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Instabilities of clock spin glasses in a magnetic field

F D Nobre† and D Sherrington

Department of Physics, Imperial College of Science and Technology, London SW7 2BZ, UK

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Abstract. We investigate the onset of replica symmetry breaking for the p-state clock spin glasses in a magnetic field. The $p = 3$ case shows a curious behaviour, the instability being of the Gabay-Toulouse type but with a different exponent; the results are non-invariant under reflection of the field ($\boldsymbol{h} \to -\boldsymbol{h}$). For $p = 4$, the instability is of the Almeida-Thouless type (Ising-like), despite the fourfold symmetry of the spin variable. For $p \geqslant 5$, the absence of reflection symmetry on the spin variable is irrelevant for p odd and the behaviour is XY-like.

1. Introduction

Spin glasses continue to be a field of strong interest where new applications and open questions appear frequently (for recent reviews see Binder and Young 1986, van Hemmen and Morgenstern 1983, 1986, Mézard *et al* 1987). Systems where the spin variable does not possess reflection symmetry, such as Potts and quadrupolar glasses (Goldbart and Sherrington 1985, Goldbart and Elderfield 1985, Gross *et al* 1985), can present very different critical behaviours from the well established m-vector spin glasses. In particular, in the above examples for zero magnetic field, the Parisi function (Parisi 1979) changes drastically as a direct consequence of the absence of reflection symmetry on the spin variable. Whether this effect is a peculiarity of Potts and quadrupolar glasses only, or if it plays an important role in other systems, is not known.

It is interesting, then, to study a p-state clock spin-glass model which can be seen as an XY model in an infinite p-fold anisotropy field. As far as the Parisi solution is concerned, the $p = 3$ case presents the 'anomaly' already observed for Potts models (indeed the $p = 3$ clock and three-state Potts models are isomorphic), but all other clock glasses behave in the conventional way, presenting a monotonically increasing function followed by a plateau (Nobre and Sherrington 1986). The investigation of the effects of a finite magnetic field in such a system is the main purpose of this paper. The onset of replica symmetry breaking for the two extremum cases, $p = 2$ (de Almeida and Thouless 1978) and $p = \infty$ (Gabay and Toulouse 1981, Cragg *et al* 1982), appear in very different ways, defining two distinct universality classes. We show that the $p = 3$ case is again peculiar, lying in a different universality class than the ones mentioned above, and that the results change under reflection of the magnetic field ($\boldsymbol{h} \to -\boldsymbol{h}$). The $p = 4$ case lies in the same class as $p = 2$ and, despite the fourfold symmetry of the spin variable, a small magnetic field induces the spin-glass order to twofold symmetric. Finally we show that all $p \geqslant 5$ clock glasses are XY-like. Therefore, similar to what

† On leave from Departamento de Física Teórica e Experimental da UFRN, 59 000 Natal, RN, Brazil.

happened to the Parisi function (Nobre and Sherrington 1986), the reflection symmetry plays a crucial role for $p=3$ but is qualitatively irrelevant for all other odd-state clock glasses.

2. The general p-state clock spin glass

A p-state clock model in a field is defined by

$$H=-\sum_{(ij)} J_{ij}\boldsymbol{S}_i\cdot\boldsymbol{S}_j-\boldsymbol{h}\cdot\sum_i \boldsymbol{S}_i \tag{2.1}$$

where the $\boldsymbol{S}_i$ are unit vectors restricted to p equally angularly spaced orientations in a plane, which can be seen as realisations of XY spin variables in a p-fold anisotropy field of infinite strength. In the infinite-ranged spin-glass version (Sherrington and Kirkpatrick 1975) the first summation is over all pairs (ij) with the J_{ij} quenched random couplings distributed according to the probability

$$P(J_{ij})=(N/2\pi J^2)^{1/2}\exp(-NJ_{ij}^2/2J^2). \tag{2.2}$$

Let us work within a representation in which the components of $\boldsymbol{S}_i$ are

$$S_{ix}=\cos\theta_i \qquad S_{iy}=\sin\theta_i \tag{2.3a}$$

$$\theta_i=\frac{2\pi}{p}k_i \qquad (k_i=0,1,\ldots,(p-1)). \tag{2.3b}$$

For a magnetic field in the x direction ($\boldsymbol{h}=h\hat{x}$), applying the standard replica trick (Edwards and Anderson 1975), one obtains the free energy per spin in the thermodynamic limit ($N\to\infty$) as the extremal problem

$$\beta f=\lim_{n\to 0}\frac{1}{n}\min\{g(R^\alpha,Q_{xx}^{\alpha\beta},Q_{yy}^{\alpha\beta})\}. \tag{2.4}$$

The functional $g(R^\alpha,Q_{xx}^{\alpha\beta},Q_{yy}^{\alpha\beta})$ is given by

$$g(R^\alpha,Q_{xx}^{\alpha\beta},Q_{yy}^{\alpha\beta})=-\tfrac{1}{8}n(\beta J)^2+\tfrac{1}{2}(\beta J)^2\sum_\alpha (R^\alpha)^2+\tfrac{1}{4}(\beta J)^2\sum_{\alpha\beta}{}'[(Q_{xx}^{\alpha\beta})^2+(Q_{yy}^{\alpha\beta})^2]-\ln\operatorname{Tr}\exp\{H_{\text{eff}}\} \tag{2.5a}$$

where

$$H_{\text{eff}}=(\beta J)^2\sum_\alpha R^\alpha[(S_x^\alpha)^2-\tfrac{1}{2}]+\tfrac{1}{2}(\beta J)^2\sum_{\alpha\beta}{}'(Q_{xx}^{\alpha\beta}S_x^\alpha S_x^\beta+Q_{yy}^{\alpha\beta}S_y^\alpha S_y^\beta)+\beta h\sum_\alpha S_x^\alpha . \tag{2.5b}$$

The spins and trace are single site and $\Sigma'_{\alpha\beta}$ denotes a sum over different replicas, $\alpha\neq\beta$. R^α is a quadrupolar parameter given by

$$R^\alpha=\langle(S_x^\alpha)^2\rangle-\tfrac{1}{2} \tag{2.6a}$$

while $Q_{xx}^{\alpha\beta}$ and $Q_{yy}^{\alpha\beta}$ are, respectively, the usual spin-glass parameters parallel and perpendicular to the external field:

$$Q_{xx}^{\alpha\beta}=\langle S_x^\alpha S_x^\beta\rangle \qquad Q_{yy}^{\alpha\beta}=\langle S_y^\alpha S_y^\beta\rangle \qquad \alpha\neq\beta. \tag{2.6b}$$

In each case the $\langle\ \rangle$ bracket denotes thermal averaging with respect to H_{eff}. For $p=2$, our model reduces to the well known Ising spin glass of Sherrington and Kirkpatrick (1975) for which R^α and $Q_{yy}^{\alpha\beta}$ are zero. In the following discussion, we shall restrict ourselves to $p>2$.

Within the replica symmetric approximation, $R^\alpha = R$, $Q^{\alpha\beta}_{xx} = Q_{xx}$, $Q^{\alpha\beta}_{yy} = Q_{yy}$, any average in the replica space in the limit $n \to 0$ is related to a disorder-averaged product of thermodynamic averages (Kirkpatrick and Sherrington 1978):

$$\lim_{n\to 0}\Big\langle \underbrace{(S^\alpha_x)^j \ldots (S^\beta_x)^j}_{q\ \text{different replicas}} \underbrace{(S^\gamma_y)^k \ldots (S^\delta_y)^k}_{r\ \text{different replicas}} \underbrace{(S^\rho_x)^g (S^\rho_y)^m \ldots (S^\sigma_x)^g (S^\sigma_y)^m}_{t\ \text{different replicas}} \Big\rangle$$

$$= [\langle S^j_x\rangle^q_{\mathrm{T}} \langle S^k_y\rangle^r_{\mathrm{T}} \langle S^g_x S^m_y\rangle^t_{\mathrm{T}}]_{\mathrm{av}} \qquad \text{(all replica indices different)}$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\mathrm{d}u\,\mathrm{d}v}{2\pi} \exp(-u^2/2)\exp(-v^2/2)$$

$$\times \left(Z^{-1}\frac{\partial^j Z}{\partial a^j_x}\right)^q \left(Z^{-1}\frac{\partial^k Z}{\partial a^k_y}\right)^r \left(Z^{-1}\frac{\partial^{g+m} Z}{\partial a^g_x\,\partial a^m_y}\right)^t \tag{2.7a}$$

where

$$Z = \mathrm{Tr}\exp(bS^2_x + a_x S_x + a_y S_y) \tag{2.7b}$$

$$a_x = \beta J Q^{1/2}_{xx} u + \beta h \qquad a_y = \beta J Q^{1/2}_{yy} v \qquad b = \tfrac{1}{2}(\beta J)^2 (2R - Q_{xx} + Q_{yy}). \tag{2.7c}$$

Now $\langle\ \rangle_{\mathrm{T}}$ denotes a thermodynamic average with respect to H and $[\ ]_{\mathrm{av}}$ denotes a quenched average over the $\{J_{ij}\}$ distribution.

One question of the greatest importance is the stability of the replica symmetric solution (de Almeida and Thouless 1978). As is well known for the m-vector spin glasses (Gabay and Toulouse 1981, Cragg *et al* 1982), a critical line in the h-T plane occurs, below which replica symmetry is unstable. For small h, this line looks in general like

$$\varepsilon \sim h^{\psi} \qquad \varepsilon = (T_{\mathrm{g}} - T)/T_{\mathrm{g}} \tag{2.8}$$

where T_{g} is the spin-glass critical temperature in zero field.

For two particular limits of the p-state clock glass the above behaviour is found but the critical exponent ψ falls into different universality classes:

(*a*) $p = 2$ (Ising spin glass) (de Almeida and Thouless 1978): in this case, the critical line is a direct consequence of replica symmetry breaking and $\psi = \frac{2}{3}$;

(*b*) $p = \infty$ (*XY* spin glass): the ordering of the transverse degrees of freedom takes place at a critical line for which $\psi = 2$ (Gabay and Toulouse 1981), and below this critical line replica symmetry is unstable (Cragg *et al* 1982).

The main purpose of this paper is to investigate how this critical line changes as we interpolate between these two limits.

In order to do this, we turn to the stability analysis in a de Almeida-Thouless fashion by taking fluctuations around the replica symmetric solutions ($R^\alpha = R + \omega^\alpha$; $Q^{\alpha\beta}_{xx} = Q_{xx} + \eta^{\alpha\beta}$; $Q^{\alpha\beta}_{yy} = Q_{yy} + \varphi^{\alpha\beta}$; $\alpha \neq \beta$). The derivation of Cragg *et al* (1982) can be reproduced straightforwardly for the p-state clock glass to give

$$(1 - \chi^{(2)}_{xx} - \lambda)(1 - \chi^{(2)}_{yy} - \lambda) - (\chi^{(2)}_{xy})^2 = 0 \tag{2.9a}$$

where

$$\chi^{(2)}_{\mu\nu} = (\beta J)^2 [(\langle S_\mu S_\nu\rangle_{\mathrm{T}} - \langle S_\mu\rangle_{\mathrm{T}}\langle S_\nu\rangle_{\mathrm{T}})^2]_{\mathrm{av}} \qquad (\mu, \nu = x, y). \tag{2.9b}$$

The 'correlation functions' $\chi^{(2)}_{\mu\nu}$ are to be evaluated in the replica symmetric approximation and stability requires all eigenvalues λ to be positive.

For large h, by solving equations (2.9), the softening to zero of λ is satisfied by small T and one gets the line

$$\frac{T}{J}=\frac{a_p}{(2\pi)^{1/2}}\exp(-h^2/2J^2) \qquad (T\sim 0) \tag{2.10}$$

which is valid in general for any value of p. The coefficient a_p is a number depending only on the value of p:

$$a_p=\int_{-\infty}^{\infty}\mathrm{d}x\left[\left(H_p^{-1}\frac{\mathrm{d}^2H_p}{\mathrm{d}x^2}\right)^2-2H_p^{-1}\frac{\mathrm{d}^2H_p}{\mathrm{d}x^2}\left(H_p^{-1}\frac{\mathrm{d}H_p}{\mathrm{d}x}\right)^2+\left(H_p^{-1}\frac{\mathrm{d}H_p}{\mathrm{d}x}\right)^4\right] \tag{2.11a}$$

where

$$H_p(x)=\sum_{k=0}^{p-1}\exp\left(x\cos\frac{2\pi k}{p}\right). \tag{2.11b}$$

It can easily be evaluated analytically for $p=2, 4$, or numerically for any p and typical values are listed in table 1.

For small h, however, different values of p lead to distinct behaviour with different values for the exponent ψ and this will concern the analysis which follows.

Table 1. The coefficient a_p (equations (2.10) and (2.11)) for several values of p; it oscillates for small p, but converges to a constant value as p gets large.

p	2	3	4	5	6	10	12	15
a_p	$\frac{4}{3}$	0.562	$\frac{2}{3}$	0.571	0.583	0.571	0.571	0.571

3. $p=3$

This is identical to the three-state Potts glass. It is well known for the isotropic case ($h=0$) that the absence of reflection symmetry on the spin variable plays a crucial role in this case, radically changing the critical behaviour (Gross *et al* 1985, Goldbart and Elderfield 1985, Nobre and Sherrington 1986). Special properties can also be noticed for the case $h\neq 0$.

For small h, by solving equations (A2) (see the appendix), one gets the critical line associated with the transverse spin-glass freezing:

$$\varepsilon=\pm\tfrac{1}{2}(4\mp\alpha_\pm)|h|/J+(13\mp\tfrac{11}{2}\alpha_\pm+\tfrac{7}{4}\alpha_\pm^2-\tfrac{1}{2}\beta_\pm)h^2/J^2+\mathrm{O}(h^3/J^3) \tag{3.1}$$

at which†

$$R=h/J+\mathrm{O}(h^3/J^3) \tag{3.2a}$$

$$Q_{xx}=\alpha_\pm|h|/J+\beta_\pm h^2/J^2+\mathrm{O}(h^3/J^3). \tag{3.2b}$$

† The quadrupolar parameter turns out be a magnetisation for $p=3$ (Elderfield and Sherrington 1983). Trivially one has

$$R=[\langle S_x^2\rangle_T]_{av}-\tfrac{1}{2}=\left[\left\langle\cos^2\frac{2\pi k}{3}-\frac{1}{2}\right\rangle_T\right]_{av}=\frac{1}{2}\left[\left\langle\cos\frac{4\pi k}{3}\right\rangle_T\right]_{av}=\tfrac{1}{2}[\langle S_x\rangle_T]_{av} \qquad (k=0,1,2).$$

In the equations above

$$\alpha_\pm = \tfrac{1}{9}(2\sqrt{34} \pm 8) \qquad (\alpha_+ \simeq 2.18,\ \alpha_- \simeq 0.41) \tag{3.3a}$$

$$\beta_\pm = \frac{\pm 96 + 104\alpha_\pm \mp 222\alpha_\pm^2 + 53\alpha_\pm^3}{18\alpha_\pm \mp 16} \qquad (\beta_+ \simeq -7.88,\ \beta_- \simeq -0.57) \tag{3.3b}$$

and the upper (lower) signs refer to $h > 0$ ($h < 0$).

In contrast to the m-vector spin glass, the results vary under the operation $\boldsymbol{h} \rightarrow -\boldsymbol{h}$ for small $|h|$ and, in particular, the parallel spin-glass parameter Q_{xx} obeys the inequality:

$$Q_{xx}(h>0) > Q_{xx}(h<0) \tag{3.4}$$

whereas the transverse spin-glass freezing temperature T_f:

$$T_f = T_g[1 \mp \tfrac{1}{2}(4 \mp \alpha_\pm)|h|/J - (13 \mp \tfrac{11}{2}\alpha_\pm + \tfrac{7}{4}\alpha_\pm^2 - \tfrac{1}{2}\beta_\pm)h^2/J^2 + O(h^3/J^3)] \tag{3.5}$$

satisfies

$$T_f(h>0) < T_g < T_f(h<0). \tag{3.6}$$

From equation (3.1) one gets that, for $h \geqslant 0$, $\varepsilon = 0$ only if $h = 0$. However, for $h < 0$, one gets two solutions with $\varepsilon = 0$, namely

$$|h|/J = 0 \qquad \text{or} \qquad |h|/J \simeq 0.14 \tag{3.7}$$

providing the small $|h|$ behaviour shown in figure 1.

Thus, the operation $\boldsymbol{h} \rightarrow -\boldsymbol{h}$ tends to weaken the parallel spin-glass ordering, while enhancing the perpendicular ordering. This is reflected in the change of sign of the parameter R and is a direct consequence of the absence of reflection symmetry in the spin variable. The fact that R is negative for $h < 0$ makes the transition in Q_{yy} more Ising-like, in contrast to the usual transition found for the m vectors. Since the Ising spin-glass transition temperature is greater than the one for the XY spin glass, one expects the increase in T_f as given by equation (3.6).

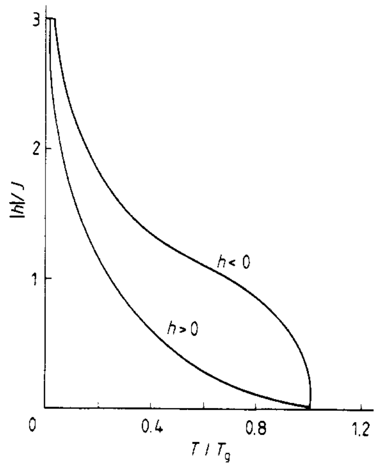

Figure 1. Phase diagram of the three-state clock spin glass in a magnetic field h. The lower line is for $h > 0$ and its shape is much like the de Almeida-Thouless line. The upper line is for $h < 0$ and for small $|h|$ one has $T_f > T_g$. In either case, the transition is of the Gabay-Toulouse type, signalling the transverse spin-glass ordering; below each line, replica symmetry is unstable.

For T just below T_f, equations (2.9) can be solved and the lowest eigenvalue is

$$\lambda = -\frac{1}{8-\alpha_\pm^2}(2\alpha_\pm^2 Q_{yy}^2/Q_{xx} + (24 \pm 16\alpha_\pm - 15\alpha_\pm^2)Q_{yy}^2) + \ldots \qquad (h \neq 0) \qquad (3.8)$$

where Q_{yy} grows as

$$Q_{yy} = \tfrac{1}{2}\varepsilon \mp |h|/J + \tfrac{1}{4}Q_{xx} \qquad (3.9)$$

and again the upper (lower) signs refer to $h > 0$ ($h < 0$). Then, replica symmetry becomes unstable at an even lower order than the usual m-vector glasses for which the instability occurs at order Q_{yy}^2 on λ. For the three-state clock spin glass in a magnetic field this instability occurs at order $\varepsilon^2 J/|h|$, or, using (3.1), at order ε. Such a lowering in the order of the instability has already been noticed for the case of a Potts glass in zero magnetic field (Elderfield and Sherrington 1983).

For large $|h|$, the transverse spin-glass transition is given by (2.10) for both h positive and negative. This is expected physically since, for either $h > 0$ (XY-like transition) (Gabay and Toulouse 1982) or $h < 0$ (Ising-like transition) (de Almeida and Thouless 1978), the large $|h|$ behaviour is the same.

Therefore, the three-state clock glass lies in a different universality class than the Ising and XY spin glasses, obeying an equation like (2.8) but with exponent $\psi = 1$. The absence of reflection symmetry on the spin variable plays a crucial role for small $|h|$, where the inversion of the field drastically changes the critical line, but becomes irrelevant for large $|h|$. The critical lines for both $h > 0$ and $h < 0$ are shown in figure 1; they signal the transverse spin-glass ordering and, below them, replica symmetry is broken.

4. $p = 4$

This is a very special case. As can be seen in the appendix, the critical temperature associated with the quadrupolar parameter R is the same as for the spin glass parameters Q_{xx} and Q_{yy}, even in zero field. This suggests that the four-state clock spin glass may present quadrupolar ordering even in zero field; this is discussed in detail in the accompanying paper (Nobre *et al* 1989) and here we shall concern ourselves with the onset of replica symmetry breaking in the presence of a field.

Let us introduce Ising variables τ_i, σ_i ($=\pm 1$) related to the S_i by

$$S_{ix} = \tfrac{1}{2}(\tau_i + \sigma_i) \qquad S_{iy} = \tfrac{1}{2}(\tau_i - \sigma_i) \qquad (4.1)$$

by means of which the Hamiltonian in equation (2.1), for the magnetic field in the x direction ($\boldsymbol{h} = h\hat{x}$), may be rewritten as

$$H = -\sum_{(ij)} \tilde{J}_{ij}(\tau_i\tau_j + \sigma_i\sigma_j) - \tilde{h}\sum_i (\tau_i + \sigma_i) \qquad (4.2a)$$

$$\{\tilde{J}_{ij}\} = \tfrac{1}{2}\{J_{ij}\} \qquad \tilde{h} = \tfrac{1}{2}h. \qquad (4.2b)$$

Therefore, the four-state clock model is equivalent to two independent Ising models, each with exchange interactions and magnetic field rescaled by a factor of one-half with respect to those of the original clock model.

In a similar way, the parameters in equations (2.6) may be re-expressed as

$$R^\alpha = \tfrac{1}{2}\langle \tau^\alpha \sigma^\alpha \rangle \qquad (4.3a)$$

$$Q_{xx}^{\alpha\beta} = \tfrac{1}{4}(\langle \tau^\alpha \tau^\beta \rangle + 2\langle \tau^\alpha \sigma^\beta \rangle + \langle \sigma^\alpha \sigma^\beta \rangle) \qquad (\alpha \neq \beta) \qquad (4.3b)$$

$$Q_{yy}^{\alpha\beta} = \tfrac{1}{4}(\langle \tau^\alpha \tau^\beta \rangle - 2\langle \tau^\alpha \sigma^\beta \rangle + \langle \sigma^\alpha \sigma^\beta \rangle) \qquad (\alpha \neq \beta). \qquad (4.3c)$$

Nobre *et al* (1989) have shown that this model is indeed 'collinear' in the sense that an infinitesimal field suffices to orient all the spins along the same axis (that of the field). This can easily be seen for the replica symmetric case where pure states are related only by global symmetry operations; because the τ and σ are completely equivalent, for a small symmetry breaking field in the x direction:

$$\langle \tau_i \rangle_T = \langle \sigma_i \rangle_T \qquad (4.4)$$

and from that follows

$$2R = Q_{xx} = [\langle \tau \rangle_T^2]_{av} \qquad Q_{yy} = 0. \qquad (4.5)$$

However, by introducing replica symmetry breaking, many pure states unrelated by symmetry appear and fluctuations from average collinearity can occur (Nobre *et al* 1988).

The onset of replica symmetry breaking can be obtained by solving (2.9), either in the S_i or in the τ_i, σ_i representations, and one gets, as expected, a de Almeida–Thouless line, taking into account the proper rescalings:

$$\varepsilon^3 = 6(h/J)^2 \qquad (h \text{ small}) \qquad (4.6a)$$

$$T/J = \tfrac{2}{3}(2\pi)^{-1/2} \exp(-h^2/2J^2) \qquad (h \text{ large}). \qquad (4.6b)$$

The Gabay–Toulouse type of behaviour, where the ordering of the transverse degrees of freedom takes place already in the replica symmetric space, cannot occur for this case as a direct consequence of equation (4.5). Despite the fourfold symmetry of the spin variable, a small magnetic field suffices to set the spin glass order to twofold symmetric.

5. $p \geq 5$

For the isotropic case ($h = 0$), $p_c = 5$ is the clock dimension at and above which the effect of the absence of reflection symmetry on the spin variable becomes irrelevant and the critical behaviour is XY-like (Nobre and Sherrington 1986). The same happens for the case $h \neq 0$ where corrections due to this effect appear only as higher-order terms in the perturbation expansion. Solving equations (A2) to leading order, one gets the critical line associated with the transverse spin-glass freezing:

$$T_f = T_g[1 - \tfrac{7}{16}(h/J)^2] \qquad (5.1a)$$

close to which

$$R = \tfrac{1}{4}(h/J)^2 \qquad Q_{xx} = \tfrac{1}{2}|h|/J. \qquad (5.1b)$$

The stability analysis can now be done by solving equations (2.9); for T just below T_f and small h, the lowest eigenvalue is given by

$$\lambda = -3Q_{yy}^2 - \tfrac{1}{8}(|h|/J)Q_{yy}^2 + O((h^2/J^2)Q_{yy}^2) \qquad (5.2)$$

which is negative, signalling instability. For large h, the onset of replica symmetry breaking is given by equation (2.10). Therefore, the clock glasses for $p \geq 5$ all lie in the same universality class as the XY spin glass (Gabay and Toulouse 1981, Cragg *et al* 1982).

6. Conclusion

By studying the p-state clock spin glass in a magnetic field we found that the $p=3$ case (three-state Potts) is very peculiar. The onset of replica symmetry breaking is of the Gabay–Toulouse type but with a different exponent. The results depend on the sign of the magnetic field and, in particular, $h>0$ ($h<0$) enhances the parallel (perpendicular) spin-glass ordering. For $p=4$, the critical behaviour is Ising-like despite the fourfold symmetry of the spin variable. A de Almeida–Thouless type of instability results in this case. For $p \geqslant 5$, the effects due to the absence of reflection symmetry on the spin variable appear only as higher-order terms in a perturbation expansion, and the dominant behaviour is XY-like.

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Appendix. Self-consistent equations in a perturbative form

In order to study the phase transition in the h–T plane, it is necessary to expand the self-consistent equations for the order parameters perturbatively. Let us then write equations (2.6) in the replica symmetric approximation (see equations (2.7)):

$$R = [\langle S_x^2\rangle_{\rm T}]_{\rm av} - \tfrac{1}{2} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{{\rm d}u\,{\rm d}v}{2\pi} \exp(-u^2/2)\exp(-v^2/2)\left(Z^{-1}\frac{\partial^2 Z}{\partial a_x^2}\right) - \frac{1}{2} \tag{A1a}$$

$$Q_{xx} = [\langle S_x\rangle_{\rm T}^2]_{\rm av} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{{\rm d}u\,{\rm d}v}{2\pi} \exp(-u^2/2)\exp(-v^2/2)\left(Z^{-1}\frac{\partial Z}{\partial a_x}\right)^2 \tag{A1b}$$

$$Q_{yy} = [\langle S_y\rangle_{\rm T}^2]_{\rm av} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{{\rm d}u\,{\rm d}v}{2\pi} \exp(-u^2/2)\exp(-v^2/2)\left(Z^{-1}\frac{\partial Z}{\partial a_y}\right)^2 . \tag{A1c}$$

Near the point where h is small and T close to T_g, R, Q_{xx}, and Q_{yy} are all small and equations (A1) can be expanded in power series to give

$$\alpha_1 + 2\alpha_2 R - \delta_2 Q_{xx} + 3\alpha_3 R^2 - \varepsilon_3 Q_{xx}^2 - \zeta_3 Q_{yy}^2 - 2\eta_3 R Q_{xx} + 4\alpha_4 R^3 - 2\zeta_4 R Q_{xx}^2$$
$$- \eta_4 Q_{xx}^3 - 3\kappa_4 R^2 Q_{xx} - 2\lambda_4 R Q_{yy}^2 - \mu_4 Q_{yy}^3 - \nu_4 Q_{xx} Q_{yy}^2 + \ldots = 0 \tag{A2a}$$

$$\beta_1 + 2\beta_2 Q_{xx} + \delta_2 R - 3\beta_3 Q_{xx}^2 + \delta_3 Q_{yy}^2 + 2\varepsilon_3 R Q_{xx} + \eta_3 R^2 + 4\beta_4 Q_{xx}^3 + 2\delta_4 Q_{xx} Q_{yy}^2$$
$$+ \varepsilon_4 Q_{yy}^3 + 2\zeta_4 R^2 Q_{xx} + 3\eta_4 R Q_{xx}^2 + \kappa_4 R^3 + \nu_4 R Q_{yy}^3 + \ldots = 0 \tag{A2b}$$

$$2\gamma_2 Q_{yy} - 3\gamma_3 Q_{yy}^2 + 2\delta_3 Q_{xx} Q_{yy} + 2\zeta_3 R Q_{yy} + 4\gamma_4 Q_{yy}^3 + 2\delta_4 Q_{xx}^2 Q_{yy} + 3\varepsilon_4 Q_{xx} Q_{yy}^2$$
$$+ 2\lambda_4 R^2 Q_{yy} + 3\mu_4 R Q_{yy}^2 + 2\nu_4 R Q_{xx} Q_{yy} + \ldots = 0. \tag{A2c}$$

The coefficients in equations (A2) are given by

$$\alpha_1 = \tfrac{1}{2}(\beta J)^2[\delta_{2,p} + \tfrac{1}{2}\beta h \delta_{3,p} + \tfrac{1}{8}(\beta h)^2(1 - \delta_{2,p} + \delta_{4,p}) + \tfrac{1}{48}(\beta h)^3(-3\delta_{3,p} + \delta_{5,p})] + \mathrm{O}(h^4) \tag{A3a}$$

$$\beta_1 = \tfrac{1}{8}(\beta J)^2(\beta h)^2(1 + 3\delta_{2,p} + \tfrac{1}{2}\beta h \delta_{3,p}) + \mathrm{O}(h^4) \tag{A3b}$$

$$\alpha_2 = \tfrac{1}{2}(\beta J)^2\{\tfrac{1}{8}(\beta J)^2[(1 - \delta_{2,p} + \delta_{4,p}) + \tfrac{1}{2}\beta h(\delta_{3,p} + \delta_{5,p}) + \tfrac{1}{8}(\beta h)^2(-3\delta_{3,p} + \delta_{6,p})] - 1\} + \mathrm{O}(h^3) \tag{A4a}$$

$$\beta_2 = \tfrac{1}{4}(\beta J)^2\{\tfrac{1}{4}(\beta J)^2[(1 + 3\delta_{2,p}) + \beta h \delta_{3,p} + \tfrac{1}{4}(\beta h)^2(-7 - 57\delta_{2,p} + \delta_{3,p} + \delta_{4,p})] - 1\} + \mathrm{O}(h^3) \tag{A4b}$$

$$\gamma_2 = \tfrac{1}{4}(\beta J)^2\{\tfrac{1}{4}(\beta J)^2[(1 - \delta_{2,p}) - \beta h \delta_{3,p} + \tfrac{1}{4}(\beta h)^2(-1 + \delta_{2,p} + \delta_{3,p} - \delta_{4,p})] - 1\} + \mathrm{O}(h^3) \tag{A4c}$$

$$\delta_2 = \tfrac{1}{8}(\beta J)^4 \beta h[\delta_{3,p} + \tfrac{1}{4}\beta h(2 - 2\delta_{2,p} + \delta_{3,p} + 2\delta_{4,p})] + \mathrm{O}(h^3) \tag{A4d}$$

$$\alpha_3 = \tfrac{1}{192}(\beta J)^6[(\delta_{3,p} + \delta_{6,p}) + \tfrac{1}{2}\beta h(-3\delta_{3,p} + \delta_{5,p} + \delta_{7,p})] + \mathrm{O}(h^2) \tag{A5a}$$

$$\beta_3 = \tfrac{1}{192}(\beta J)^6[8 + 56\delta_{2,p} - (1 - 21\beta h)\delta_{3,p}] + \mathrm{O}(h^2) \tag{A5b}$$

$$\gamma_3 = \tfrac{1}{24}(\beta J)^6(1 - \delta_{2,p} - \tfrac{3}{2}\beta h \delta_{3,p}) + \mathrm{O}(h^2) \tag{A5c}$$

$$\delta_3 = \tfrac{1}{64}(\beta J)^6(1 - \beta h)\delta_{3,p} + \mathrm{O}(h^2) \tag{A5d}$$

$$\varepsilon_3 = \tfrac{1}{64}(\beta J)^6[2(1 - \delta_{2,p} + \delta_{4,p}) + \beta h(-6\delta_{3,p} + \delta_{5,p})] + \mathrm{O}(h^2) \tag{A5e}$$

$$\zeta_3 = \tfrac{1}{64}(\beta J)^6[2(-1 + \delta_{2,p} - \delta_{4,p}) + \beta h(\delta_{3,p} - \delta_{5,p})] + \mathrm{O}(h^2) \tag{A5f}$$

$$\eta_3 = \tfrac{1}{64}(\beta J)^6[2\delta_{3,p} + \beta h(3\delta_{3,p} + \delta_{5,p})] + \mathrm{O}(h^2) \tag{A5g}$$

$$\alpha_4 = \tfrac{1}{3072}(\beta J)^8(-3 + 3\delta_{2,p} - 13\delta_{4,p} + \delta_{8,p}) + \mathrm{O}(h) \tag{A6a}$$

$$\beta_4 = \tfrac{1}{3072}(\beta J)^8(117 + 2059\delta_{2,p} - 48\delta_{3,p} - 17\delta_{4,p}) + \mathrm{O}(h) \tag{A6b}$$

$$\gamma_4 = \tfrac{1}{3072}(\beta J)^8(117 - 117\delta_{2,p} - 17\delta_{4,p}) + \mathrm{O}(h) \tag{A6c}$$

$$\delta_4 = \tfrac{1}{512}(\beta J)^8(5 - 5\delta_{2,p} - 8\delta_{3,p} + 7\delta_{4,p}) + \mathrm{O}(h) \tag{A6d}$$

$$\varepsilon_4 = -\tfrac{1}{32}(\beta J)^8\delta_{3,p} + \mathrm{O}(h) \tag{A6e}$$

$$\zeta_4 = \tfrac{1}{256}(\beta J)^8(1 - \delta_{2,p} - 7\delta_{3,p} + 3\delta_{4,p} + \delta_{6,p}) + \mathrm{O}(h) \tag{A6f}$$

$$\eta_4 = \tfrac{1}{128}(\beta J)^8(-4 + 4\delta_{2,p} - 3\delta_{3,p} - 4\delta_{4,p}) + \mathrm{O}(h) \tag{A6g}$$

$$\kappa_4 = \tfrac{1}{128}(\beta J)^8\delta_{3,p} + \mathrm{O}(h) \tag{A6h}$$

$$\lambda_4 = \tfrac{1}{256}(\beta J)^8(1 - \delta_{2,p} - \delta_{3,p} + 3\delta_{4,p} - \delta_{6,p}) + \mathrm{O}(h) \tag{A6i}$$

$$\mu_4 = \tfrac{1}{32}(\beta J)^8(1 - \delta_{2,p} + \delta_{4,p}) + \mathrm{O}(h) \tag{A6j}$$

$$\nu_4 = -\tfrac{1}{128}(\beta J)^8\delta_{3,p} + \mathrm{O}(h). \tag{A6k}$$

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